VI. SOLAR VARIATIONS COMPUTED FROM OBSERVATIONS AT INDEPENDENT STATIONS

Disregarding a small constant difference between mean solar constants from groups of same-day observations at Montezuma and Harqua Hala, also the artificial correlation previously mentioned, we give in Table 7 the results of the application of equations (10), (11), and (12) to the evaluation of the station errors σ_x Montezuma, σ_y Harqua Hala and σ_i possible solar variability.

For comparative purposes the 399 observations made on the same days at both stations are divided into three groups representing more or less homogeneous values. A limited number of simultaneous bolographic observations were made at Mount Wilson and Calama during the years 1918, 1919, and 1920. By combining all the values into one group, being careful to preserve the lowest possible minimum sum of squares of variations and differences, by excluding effects due to large secular changes between years and to scale differences between stations, we get the results given in the bottom line of values in Table 7 indicating a possible solar variation of 0.55 per cent, which is from two to four times the variation shown by the other data. Comparing these results with the magnitude of the station errors we see that σ_i is a function of those errors, a fact which, as pointed out in Section I, invalidates the assumption that equations (10), (11), and (12) are three simultaneous equations between three independent unknowns. The results in Table 7, therefore, must be interpreted to mean that either solar variation is nonexistent or is relatively so small that it can not be disentangled from the irregular larger variations in daily values due to errors of observation.

TABLE 7.—*Calculation of solar variations from synchronous observations at Harqua Hala or Mount Wilson, designated by subscript x and at Calama or Montezuma, designated by subscript y.*

Date	Number of observations	Solar constant E_o		Total variation T			Calculated values			$\frac{0.6745\sigma_i}{E_o}$
		$\bar{E}_x$	$\bar{E}_y$	T_x	T_y	T_{xy}	σ_x	σ_y	σ_i	
										Per cent
Oct. 4, 1920, to Mar. 31, 1922	100	1.9460	1.9467	0.0129	0.0171	0.0176	0.0096	0.0148	0.0086	0.30
Apr. 1, 1922, to July 1, 1923	106	1.9168	1.9210	.0115	.0143	.0173	.0107	.0136	.0043	.15
Aug. 1, 1923, to Nov. 30, 1924	193	1.9251	1.9231	.0108	.0139	.0156	.0091	.0126	.0058	.20
July 27, 1918, to Sept. 6, 1920	66	1.9457	1.9417	.0301	.0241	.0314	.0256	.0174	.0158	.55

VII. CONCLUSION

Final definitive evidence, especially in quantitative measures, can not be secured from observations at a single station with only one set of observing instruments. Much could be learned from check observations by wholly independent equipments maintained side by side, and it is hoped such check determinations can be secured at some station to be established in the future.

A considerable number of synchronous observations have been secured from stations in pairs, as Bassour and Mount Wilson, the latter and Calama, including Calama and Montezuma with Harqua Hala. Unfortunately, because of volcanic dust and other untoward circumstances, these synchronous values are so much affected by important accidental and systematic terrestrial and artificial causes as to more or less invalidate the evidence which these observations might show of a small possible variation, which can be entertained as real only when confirmed by future *independent* observations at other stations.

It is indicated in the text that pyrheliometer readings alone are nearly errorless values from which real solar variability can be proven and evaluated with considerable accuracy, especially if observations are secured from uniformly standardized instruments observed at several wholly independent stations in the arid regions of the earth and over as great a range of elevation as possible.

The International Commission for Solar Radiation has this subject under consideration, and the writer hopes its actions may lead to progress in this important field of geophysical science.

The presentation in this paper is offered as an example, so to speak—a preliminary survey and study. I expect to extend the investigation to many other observations thus far examined not at all or only in the most superficial way.

SMITHSONIAN SOLAR-CONSTANT VALUES

By HERBERT H. KIMBALL

[Washington, Sept. 1, 1925]

SYNOPSIS

This paper considers briefly the magnitude of errors in solar constant determinations arising from errors in the fundamental pyrheliometric readings and in their extrapolation to zero atmosphere.

The degree of correlation between solar constant determinations made nearly simultaneously at Montezuma, Chile, and Harqua Hala, Ariz., leads to the conclusion that only an insignificant part of their day-to-day variations can be attributed to some such common cause as solar variability.

INTRODUCTION

A critical study of the work during the past 20 years of Doctor Abbot and his associates in connection with determinations of the value of the solar constant leads one to a profound respect for the skill, energy, and devotion to science that is evident throughout. It is not a simple matter to obtain from measurements of solar radiation intensity made at the bottom of the sea of air the intensity of that radiation before it enters our atmosphere. This is what they have done, however, and with such precision that the published mean value of their determinations, 1.94 gram-calories per minute per square centimeter, is almost universally accepted, although this value is necessarily subject to a probable error that as yet can hardly be evaluated. That Doctor Abbot and his associates seem to recognize this is indicated by their statement that after all possible care in the standardization and intercomparison of instruments employed at Montezuma and Harqua Hala it was necessary to add a little more than 1 per cent to the solar constant determinations made at the latter station to bring them into accord with those at the former station.[1]

[1] Abbot, C. G., and Colleagues. Values of the solar constant, 1920-1922. MONTHLY WEATHER REVIEW February 1923, 51: 71-81. (See especially p. 74.)

ERRORS IN PYRHELIOMETRIC READINGS

When, however, we consider the day-to-day variation in the solar constant values, questions arise as to the extent to which these represent unavoidable errors of observation and reduction and to what extent they represent solar variability.

Some data bearing upon this question are available. For example, at the basis of all solar constant determinations are the pyrheliometric readings, and the means of series of comparative readings between different pyrheliometers given in the Annals of the Astrophysical Observatory, volume 4, pages 94–95, show an average variation in the mean ratio of the different series of a little more than one-half of 1 per cent. Or, stated in another way, the probable error in the ratio of any one of 37 series of comparative readings is ± 0.42 per cent.

For Montezuma, Doctor Abbot computes the probable error of a series of readings with one pyrheliometer of the improved silver disk type to be only ± 0.2 per cent; and since at that station two pyrheliometers are read in connection with each solar constant determination, he estimates the error of the determination due to inaccuracy in the pyrheliometric readings[2] to be only

$$\sqrt{\frac{(0.20)^2}{2}} = 0.14 \text{ per cent.}$$

This degree of accuracy can not be claimed for earlier determinations, where only one pyrheliometer, and that one of an earlier and less accurate type, was read.

In a recent publication[3] he computes the probable error of a solar constant determination at a single station to be ± 0.0065 gr. cal. per min. per cm^2, or ± 0.335 per cent; while for the mean of the determinations at two stations the probable error is given as ± 0.0046 gr. cal., or ± 0.237 per cent.

Assuming that Abbot's method of determining the probable error is correct in this case, it leads to the surprising result that the portion of the probable error attributable to errors in pyrheliometric readings outweighs that due to all other sources combined, both instrumental and atmospheric. In this connection it may be pointed out that Abbot's method of computing the probable error of solar constant values from the "average daily difference, Harqua Hala—Montezuma," is applicable only if the determinations at the two stations are entirely independent of one another.

The above does not include possible constant or secular errors in the pyrheliometric scale. It is inevitable that inaccuracies in the fundamental pyrheliometric readings will be carried into the day-to-day values of the solar constant, and will form a material part of the apparent day-to-day variations in the solar constant values.

ERRORS IN EXTRAPOLATION TO ZERO AIR MASS

We next have to consider the difficulty of extrapolating the pyrheliometric readings to zero air mass, or correcting for loss of intensity of radiation by absorption and scattering in its passage through the atmosphere. Doctor Abbot and his associates accomplish this by computing the atmospheric transmission for monochromatic radiation of about 40 different wave lengths from spectro-bolometric measurements, or by a secondary process based on spectro-bolometric measurements of the precipitable water in the atmosphere and pyranometer measurements of the brightness of the sky around the sun. Neither of these processes can give results that are absolutely correct. Without discussing the causes that lead to errors we have only to examine the individual determinations at the two stations Montezuma and Harqua Hala to see that this is so.[4]

In Figure 1 I have selected for this examination the solar constant values obtained on December 15, 1920, and June 9, September 27, October 3, October 7, November 21, December 2, and December 9, 1921.[5] These are days on which at the two stations at least six determinations of the value of the solar constant were obtained, and were given a rating of S or $S-$ (satisfactory or nearly satisfactory). They may be taken as typical of days so rated. Under each date the first row of values is for Harqua Hala, the second for Montezuma. Open circles indicate the individual determinations, closed circles the weighted mean value for each station and crosses the means for each day derived from the determinations at both stations. The probable error of the individual determinations on different dates varies between $\pm 0.23\%$ and $\pm 0.74\%$.

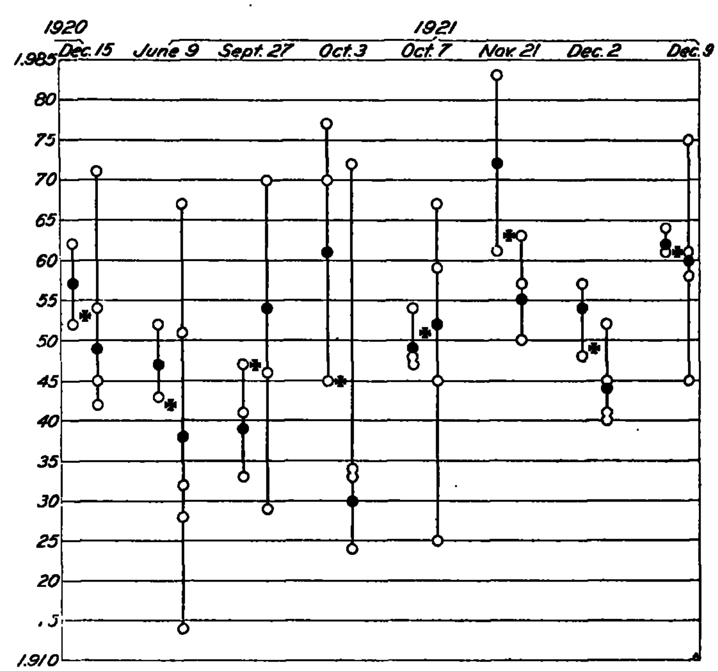

FIG. 1.—Individual, and mean values of solar constant determinations at Harqua Hala, Ariz., and Montezuma, Chile, on selected dates

We note the method of determining the solar constant value for a given day. The observations for each station are weighted in accordance with their reliability in the judgment of the observer and the computer, and from these weighted values the station mean is found. The mean of these two station means gives the adopted value for the day. It will be noted that on these eight particular days the difference between the extreme daily means is from 1.942 to 1.963, or 0.021 gr. cal., while the difference between the extremes of station means is from 1.930 to 1.972, or just twice as much.

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[2] Annals of the Astrophysical Observatory, v. 4, pp. 162 and 166.

[3] Solar variation and forecasting. Smithsonian Misc. Coll., vol. 77, no. 5, 1925.

[4] Abbot, C. G., and Colleagues. Provisional solar constant Values, August, 1920, to November, 1924. Smithsonian Misc. Coll. vol. 77, No. 3, Feb., 1925.

[5] See Table 8, MONTHLY WEATHER REVIEW, February, 1923, 51:78–81.

CORRELATION BETWEEN DETERMINATIONS AT TWO STATIONS

There are 398 days between October, 1920, and November, 1924, inclusive on which solar constant values obtained at both Montezuma and Harqua Hala were considered good enough to be used in obtaining the mean solar constant value for the day. These values have been divided into three groups, as follows:

October, 1920, to March, 1922, inclusive, with 99 determinations at each station; April, 1922 to July, 1923, inclusive, with 106 determinations at each station; August, 1923, to November, 1924, inclusive, with 193 determinations at each station. It will be noted that the grouping has been made in such a way as to throw into the first group most of the secular variation that appears in the solar constant values during this time, and especially during the early months of 1922.

In Figures 2 and 3 the data represent values of the solar constant as determined at the two stations on the same day. The abscissa of a dot gives the value of the Montezuma determination; the ordinate, the value of the Harqua Hala determination. Figure 2 includes all the 99 pairs of values of the first group except two in which the Harqua Hala values, 2.021 and 2.000, fall beyond the upper limit of the ordinate scale of the figure. These values, with the corresponding values for Montezuma, 1.954 and 1.968, respectively, were included in the computation of the correlation coefficient between the solar constant determinations at the two stations, and this coefficient was found to be $+0.341 \pm 0.060$. Figure 3

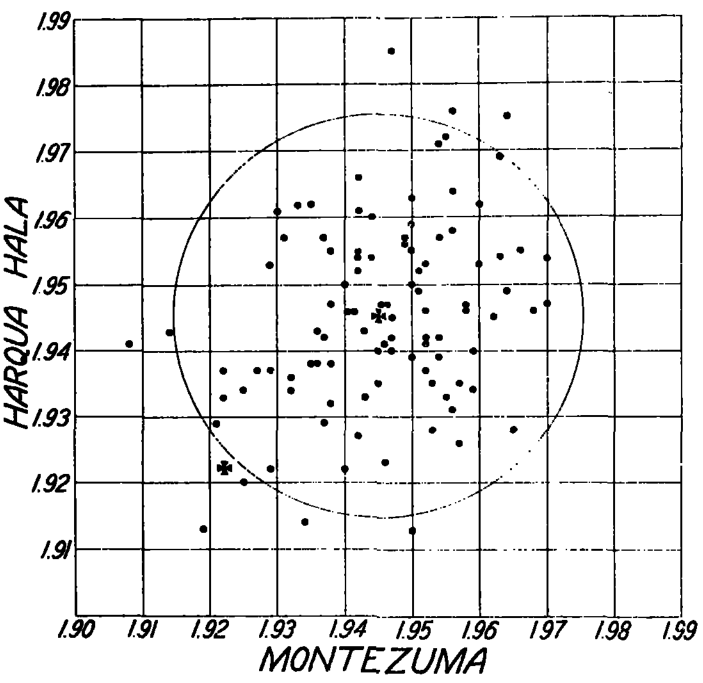

FIG. 2.—Values of the solar constant determined at Harqua Hala, Ariz., and Montezuma, Chile, on the same date, between October, 1920, and March, 1922, inclusive

includes all the 299 values of the second and third groups. The correlation coefficient for the second group is $+0.18 \pm 0.063$, and for the third group $+0.17 \pm 0.045$.

We have two methods of determining the significance of these correlation coefficients, as follows:

(1) It is generally agreed that before a correlation coefficient can begin to have significance it must exceed its probable error four fold.[6] Therefore, the coefficients for groups 2 and 3 have little or no significance, and that for Group 1 may have significance.

(2) Whipple[7] has shown that the degree of relationship between two variables is measured, not by the correlation coefficient, but by its square. This gives about 0.12 for the degree of relationship between Montezuma and Harqua Hala solar constant values of the first group. Therefore, of their standard deviations which are ± 0.013

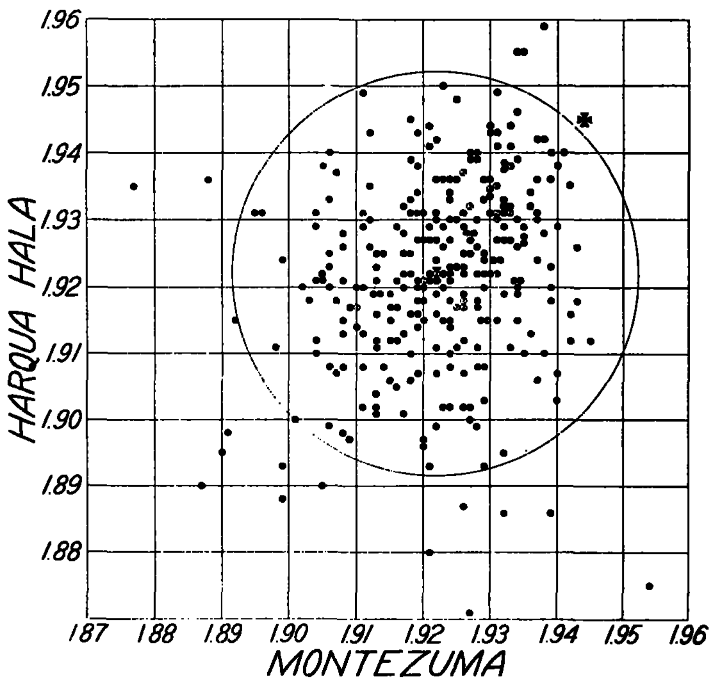

FIG. 3.—Values of the solar constant determined at Harqua Hala, Ariz., and Montezuma, Chile, on the same date, between April, 1922, and November, 1924, inclusive

and ± 0.017 gr. cal., respectively, only about ± 0.002 gr. cal. can be attributed to some such common influence as solar variability. Similarly, the extreme range of values in this group is for Harqua Hala from 2.021 to 1.913, or 0.118 gr. cal., and for Montezuma from 1.970 to 1.908, or 0.062 gr. cal. of which certainly not more than about 0.014 gr. cal. can be attributed to some such common influence as solar variability; and this is less than the secular variation indicated by the monthly mean values of the group, which have for their extremes 1.957 for January, 1921, and 1.934, for June, 1921, and March, 1922, or a range of 0.023 gr. cal.

In Groups 2 and 3, in which the standard deviations of the solar constant determinations for Montezuma are ± 0.012 and ± 0.011 gr. cal., respectively, and for Harqua Hala ± 0.014 for both groups, the square of the correlation coefficients gives approximately 0.03 for the degree of relationship between the determinations of the value of the solar constant at the two stations. Of the extreme range in values, which at Harqua Hala is from 1.958 to 1.871, or 0.088 gr. cal., and at Montezuma from 1.954 to 1.877, or 0.077 gr. cal., not over 0.003 gr. cal. can be attributed to some such common cause as solar variability, an amount which is quite negligible.

It can not be without significance that for the period April, 1922, to November, 1924, inclusive, or for two years and eight months, the correlation between the day-to-day values of the solar constant determined at

[6] Yule, G. Udny. Theory of statistics, 5th edition, London, 1919, p. 311.

[7] Whipple, F. J. W. The significance of correlation coefficients. Meteorological Magazine, Feb. 1921, 56:20-21.

the two stations indicates a degree of relationship so slight as to be negligible. During this period the range in the monthly mean values was from 1.933 to 1.907, or 0.026 gr. cal.

On Figures 2 and 3 a cross has been located within the circles that inclose most of the dots, to show the mean value of the solar constant determinations indicated by the dots. A second cross has been located at a point to indicate the mean of solar constant values represented by the dots on the other corresponding figure. These two crosses fall on a line making an angle of very nearly 45° with the axes of ordinates and abscissas, which indicates that the results as summarized above are not inconsistent with Abbot's finding of a high correlation coefficient between the means of groups of synchronous values at Montezuma and Harqua Hala, arranged in order of magnitude of the solar constant determinations at Montezuma.[8] Such a grouping eliminates very largely the day-to-day variations discussed above; and the correlations found are the result of secular changes occurring during the period covered by the three groups, and not real correlations between actual day-to-day changes free from secular variations.

Furthermore, during the period April, 1922, to November, 1924, inclusive, a solar constant value is given on 827 days, and on 299 of these days, or on 1 day out of 2.78, values obtained at both stations were included in the mean for the day. The mean value for the whole period is 1.922. On 36 days the value was less than 1.900, and on 6 of these days, or on only 1 day out of 6, was the value derived from determinations made at both stations. On 35 days the value was 1.940 or above, and on 4 of these, or on 1 day out of 8.75, the value was derived from determinations made at both stations. Thus, while more than one third of the daily values have been derived from measurements made at both stations, this is the case with only about one seventh of the values that depart from the mean by more than ±1%.

In view of the above, and for the further reason that one out of every six of these extreme values is by Abbot graded $U+$ or U (rather unsatisfactory or unsatisfactory), they are not entitled to as much weight as the more nearly average values.

Abbot holds the view that since there are more *values with large departures* than theory calls for, this is a proof of solar variability.[9] It is not unusual to find such an excess however.[10] Further, in this particular case, it has been shown above that these extreme values have not the same degree of accuracy as the remaining values. Therefore the excess in their number, which is small numerically, can not be accepted as evidence of solar variability.

It seems evident, therefore, that the day-to-day variability of the solar constant determinations, the standard deviation of which is less than ±0.70 per cent, depends largely upon whether the solar constant value is derived from determinations made at only one or at both stations; that it reflects unavoidable inaccuracies in pyrheliometric readings, and in extrapolating the readings to zero air mass; and that short-period solar variability, if it exists, falls within the limits of the probable error of the determinations.

[8] Abbot, C. G., Solar variation and forecasting, p. 20, Smithsonian Misc. Coll., vol. 77, No. 5.

[9] Solar variability and forecasting. Smithsonian Misc. Coll., vol. 77, No. 5, pp. 16–18.

[10] Brunet, David. The combination of observations. (Cambridge, 1917) p. 33.

THE PROBABLE 24-HOUR TEMPERATURE CHANGE (7 A. M. TO 7 A. M.) AT MONTGOMERY, ALA.

By JESSE W. SMITH and WELBY R. STEVENS

[Montgomery, Ala., Weather Bureau Office, May 18, 1925]

In this study the probable temperature change for the 24-hour period 7 a. m. to 7 a. m., 90th meridian time, at Montgomery, Ala., has been determined by means of the Gram-Charlier frequency curves for each month of the year, based on 1,000 observations for each month.

The temperature changes were determined from the a. m. observations as recorded on Form No. 1001–Metl., beginning with 1924 and going back far enough to include 1,000 days in each monthly distribution. Each temperature was taken to the nearest even degree before the change was computed, thus giving the change in 2° units. This was done in order to give actual changes considered in the verification of forecasts.

The Gram-Charlier curves were selected because of the relative ease with which they may be computed and their flexibility, which promised good fits in all cases. Reference to Figure 1 shows that very good fits were obtained. It is the belief of the authors that the Gram-Charlier curves are particularly well adapted to meteorological distributions, because of their capacity to take care of both skewness and excess, which are likely to be encountered in appreciable degree, especially in monthly distributions.

It seems unnecessary to give a detailed description of the method of fitting these curves. Reference is made to Fisher,[1] where a lucid explanation may be found, both of the mathematical development and practical application. However, for practical application, tables [2] will be necessary containing the values of the generating function $\varphi_0(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2:2}$ and its derivatives up to at least the fourth order.

Computations and the necessary control checks were made for each month, of which only January is shown, in Table 1, to illustrate the method.

Perhaps slightly better fits would have been obtained had the parameters been computed by the method of least squares instead of the method of semi-invariants, but the arithmetical labor entailed would have been almost prohibitive.

It is well known that in dealing with a limited series of observations the third and fourth moments, or in this case the third and fourth semi-invariants, are liable to considerable error, due to the chance presence of a few large departures. It was found in most of the calculations that this error was sufficient to cause a slowing up in the rate of increase near the tails of the curves, or in some cases enough to cause serious secondary inflections. It was found possible to eliminate this undesirable situation by neglecting, in the computation of the third and fourth semi-invariants, the observations at the tails beyond the value $z = (x - \lambda_1) : \sigma > 4$. Inasmuch as never more

[1] Fisher, Arne, Mathematical Theory of Probabilities, New York, 1923.

[2] Jørgensen, N. R., Undersøgelser over Frequensflader og Korrelation, Copenhagen, 1916.